# Number and Operations

## *Direct/Inverse Proportion*

$a$ and $b$ are directly proportional $\leftrightarrow \frac{a}{b} = k$

$a$ and $b$ are inversely proportional $\leftrightarrow ab = k$

$k$ is the proportional constant

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## *Arithmetic Sequence/Series*

$d = a_{n+1} - a_n$

$a_n = a_1 + d(n-1)$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}\big(2a_1 + d(n-1)\big)$

## *Geometric Sequence/Series*

$r = \frac{a_{n+1}}{a_n}$

$a_n = a_1 r^{n-1}$

$S_n = \frac{a_1(1-r^n)}{1-r}, r \neq 1$

$S_\infty = \frac{a_1}{1-r}, |r| < 1$

$a_n$ = value of the $n^{\text{th}}$ term, $a_1$ = value of the first term, $d$ = common difference, $r$ = common ratio, $n$ = place of a specific term, $S_n$ = sum of the first $n$ terms, and $S_\infty$ = sum of infinite terms (geometric only)

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## *Counting*

$n\mathbf{P}r =$ choosing $r$ items from a total of $n$ items, order matters (no repetition)

$n\mathbf{C}r = \binom{n}{r}$ = choosing $r$ items from a total of $n$ items, order does not matter (no repetition)

$n! = n\times(n-1)\times(n-2)\times \ldots \times3\times2\times1$

$0! = 1$

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## *Logarithms*

$\log_a x = y \leftrightarrow a^y = x$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^y) = y\log_a(x)$

$\log_a a = 1$

$\log_a 1 = 0$

$\log_a 0 = \text{undefined}$

$x = \log_a a^x$

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## *Vector*

For vector $\boldsymbol{u} = \langle u_1, u_2, u_3\rangle$ and $\boldsymbol{v} = \langle v_1, v_2, v_3\rangle$ [omit $u_3/v_3$ respectively if vector is two-dimensional]

$|\boldsymbol{u}| = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2}$ [this is called the magnitude, which is the vector's length]

$k\boldsymbol{v} = \langle kv_1, kv_2, kv_3\rangle$

$\boldsymbol{u} \pm \boldsymbol{v} = \langle u_1 \pm v_1, u_2 \pm v_2, u_3 \pm v_3\rangle$

$\boldsymbol{u} \cdot \boldsymbol{v} = u_1v_1 + u_2v_2 + u_3v_3$

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## *Matrices*

For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, determinant of $A$, $\det(A) = |A| = ad - bc$

For matrix $A$ with $m$ rows and $n$ columns (a $m$-by-$n$ matrix), and matrix $B$ with $n$ rows and $p$ columns (a $n$-by-$p$ matrix), where $m \neq n \neq p$:

$AB$ is a matrix with $m$ rows and $p$ columns [column of $A$ = row of $B$]

$BA$ is undefined [column of $B \neq$ row of $A$]

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