

# Geometry and Measurement

## *Coordinate Geometry*

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Distance [2D]} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{[3D]} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Midpoint [2D]} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{[3D]} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

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## *Polar Coordinates*

In polar form, each coordinate pair is written as  $(r, \theta)$

To convert from polar to Cartesian  $(xy)$  coordinates:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

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## *Transformation*

For  $k > 0$ :

$f(x) + k \rightarrow$  function is translated  $k$  units upward

$f(x) - k \rightarrow$  function is translated  $k$  units downward

$f(x + k) \rightarrow$  function is translated  $k$  units to the left

$f(x - k) \rightarrow$  function is translated  $k$  units to the right

$-f(x) \rightarrow$  function is reflected along the  $x$ -axis (upside-down flip)

$f(-x) \rightarrow$  function is reflected along the  $y$ -axis (left-right flip)

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## *Degree vs. Radian*

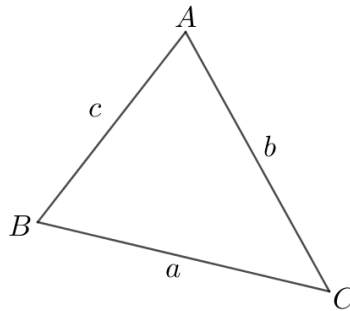
Degree to Radian:  $x^\circ \times \frac{\pi}{180^\circ} = \text{radian}$

Radian to Degree:  $x \times \frac{180^\circ}{\pi} = \text{degree}$

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## Trigonometry for Non-Right-Angle Triangle

Let  $A, B,$  and  $C$  be the angles of a triangle, and  $a, b,$  and  $c$  be the corresponding sides.



$$\text{Law of sine: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Law of cosine: } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\text{or } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

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## Trigonometric Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} / \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} / \sec \theta = \frac{1}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 / \sin^2 \theta = 1 - \cos^2 \theta / \cos^2 \theta = 1 - \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta / 1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$